

有界窄带激励下中耳系统主共振分析*

PRIMARY RESONANCE ANALYSIS OF MIDDLE EAR SYSTEM SUBJECTED TO NARROW-BAND RANDOM EXCITATION

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摘要 以中耳系统为研究对象,研究人耳在窄带随机声波激励下的非线性振动特性。根据中耳三自由度非线性模型建立非线性随机振动方程并引入正则矩阵,得到正则坐标下的非线性振动方程。采用多尺度法得到中耳系统第1阶主共振的幅频特性方程,通过分岔图和最大Lyapunov指数,发现在不同的激励下,中耳系统会产生不同的运动。导出中耳系统伊藤随机微分方程,使用矩法得到系统2阶矩的近似表达式,验证随机扰动强度对中耳系统的影响。结果表明,增大外激励,系统共振区域增大;当随机扰动强度增大时,相图极限环厚度变厚。

关键词 中耳系统 非线性 主共振 分岔 窄带激励

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Abstract Taking the middle ear system as the research object, the nonlinear vibration characteristics of the human ear under narrow-band random acoustic excitation were studied. According to the three-degree-of-freedom nonlinear model of the middle ear, the nonlinear random vibration equation was established and the regular matrix was introduced to obtain the nonlinear vibration equation in the regular coordinate. The amplitude-frequency characteristic equation of the first-order primary resonance of the middle ear system was obtained by the multi-scale method. Through the bifurcation diagram and the maximum Lyapunov index, it was found that the middle ear system produces different motions under different excitations. The Ito stochastic differential equation of the middle ear system was derived, the approximate expression of the second-order moment of the system was obtained by using the rectangular method, and the influence of the random disturbance intensity on the middle ear system was verified. The results show that the resonance area of the system increases with the increase of the external excitation. When the random disturbance intensity increases, the thickness of the limit cycle of the phase diagram becomes thicker.

Key words Middle ear system; Nonlinear; Primary resonance; Bifurcation; Narrow-band excitation

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0 引言

人类中耳主要由听小骨和软组织等构成,其中听小骨由锤骨、砧骨、镫骨等组成,软组织主要包括鼓膜、韧带及肌腱等^[1]。中耳的功能是传导声音,即中耳鼓膜受到的声波激励,通过听小骨转变为机械振动信号传递到内耳中^[2]。由于恶劣的工作环境、不良的生活习惯和遗传等因素,人们容易感染中耳

疾病,因此研究人类中耳系统生物力学对认识中耳传声机制,以及病理状态下的传声机制有重要意义^[3]。

在过去的几十年里,根据中耳结构,建立和分析了三自由度^[4]、四自由度^[5]和六自由度^[6]弹簧-阻尼-质量模型。这些模型镫骨的位移与实验结果符合程度较好,然而,这些模型都是线性模型,无法解释中耳行为的复杂性。EBRAHIMIAN A

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等^[7]通过实验测得连接镫骨和耳蜗的环形韧带的力学特性接近三次方曲线规律。RUSINEK R等^[8]⁴⁵⁻⁵⁷在中耳三自由度线性力学模型的基础上,将环形韧带非线性特性引入到人类中耳三自由度模型中,建立中耳三自由度弱非线性力学模型,并进行了实验测试,证明模型给出的结果与测试的实验结果基本一致。

在以上研究中,学者主要研究确定激励对中耳系统的影响。在实际生活中,声波激励大多数是随机的、不确定的,很多情形下可以将外部声波激励视为窄带随机激励过程^[9]。窄带随机激励的频谱主要集中在某段区间,且窄带随机激励下的系统非线性效应将更为突出,更具有研究意义^[10]。

本文应用多尺度法计算了有界窄带声波激励下第1阶主共振的幅频特性方程和系统2阶矩的近似表达式,用Runge-Kutta法计算得到系统的时间历程曲线和相图,最后分析对比随机参数对中耳系统共振的影响。本研究为中耳传声机制和病理分析提供一定的理论参考价值。

1 中耳系统非线性动力学方程

文献^[8]⁴⁵⁻⁵⁷根据中耳结构(图1),将听小骨等效为质量块,软组织的力学特性由弹簧和阻尼器表示,建立三自由度中耳系统非线性模型(图2)。该模型由3个集中质量和7个弹簧和7个阻尼器组成。锤骨、砧骨和镫骨分别用质量 m_M 、 m_I 、 m_S 表示, k_1 、 c_1 分别为鼓膜的刚度和阻尼; k_2 、 c_2 分别为锤骨前踝韧带的刚度和阻尼; k_3 、 c_3 分别为砧锤关节的刚度和阻尼; k_4 、 c_4 分别为砧骨后韧带的刚度和阻尼; k_5 、 c_5 分别为砧镫关节的刚度和阻尼; k_6 、 c_6 分别为耳蜗的刚度和阻尼;环形韧带是连接镫骨和耳蜗的软组织,具有

非线性特性 $F_7 = k_{71}x + k_{72}x^2 + k_{73}x^3$ 。其中, k_{71} 为环形韧带的线性部分刚度; k_{72} 为二次部分刚度; k_{73} 为三次部分刚度; c_7 为环形韧带的阻尼。

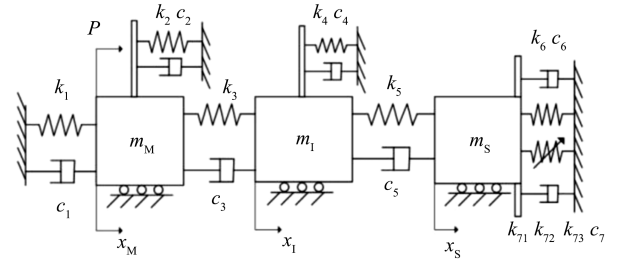


图2 三自由度中耳系统非线性模型

Fig. 2 Nonlinear model of the three-degree-of-freedom middle ear system

$p = p(t)$ 为中耳鼓膜受到的声波窄带随机激励,其表达式为

$$p(t) = p \cos [\Omega t + \gamma W(t)], p > 0$$

式中, $p(t)$ 为声波激励的幅值, Ω 为随机激励的中心频率; $W(t)$ 为单位强度的维纳过程; γ 为随机激励的扰动强度, $\gamma \geq 0$ 。

根据牛顿第二定理建立非线性动力学方程为

$$\begin{cases} m_M \ddot{x}_M + k_1 x_M + k_3 (x_M - x_I) + \\ c_1 \dot{x}_M + k_2 x_M + c_2 \dot{x}_M + c_3 (\dot{x}_M - \dot{x}_I) = \\ p \cos [\Omega t + \gamma W(t)] \\ m_I \ddot{x}_I + k_3 (x_I - x_M) + k_4 x_I + \\ k_5 (x_I - x_S) + c_3 (\dot{x}_I - \dot{x}_M) + \\ c_4 \dot{x}_M + c_5 (\dot{x}_I - \dot{x}_S) = 0 \\ m_S \ddot{x}_S + k_5 (x_S - x_I) + k_6 x_S + \\ k_{71} x_S + k_{72} x_S^2 + k_{73} x_S^3 + c_5 (\dot{x}_S - \dot{x}_I) + \\ c_6 \dot{x}_S + c_7 \dot{x}_S = 0 \end{cases} \quad (1)$$

由于中耳参数的数量级相差较大,引入无量纲参数,为

$$\begin{cases} \tau = \omega_M t, \omega_M = \sqrt{\frac{k_1}{m_M}}, \tau = \omega_M t \\ M_M = \frac{m_M}{m_M}, M_I = \frac{m_I}{m_M}, M_S = \frac{m_S}{m_M} \\ X_M = \frac{x_M}{x^*}, X_I = \frac{x_I}{x^*}, X_S = \frac{x_S}{x^*} \\ K_i = \frac{k_i}{k_1}, i = 1, 2, 3, 4, 5, 6, K_{71} = \frac{k_{71}}{k_1} \\ K_{72} = \frac{k_{72}}{k_1} x^*, K_{73} = \frac{k_{73}}{k_1} (x^*)^2 \\ C_j = \frac{c_j}{m_M \omega_M}, j = 1, 2, 3, 4, 5, 6, 7 \\ P = \frac{p}{k_1 x^*} \end{cases} \quad (2)$$

式中, $x^* = 10^{-5} m$,无量纲处理后的方程为

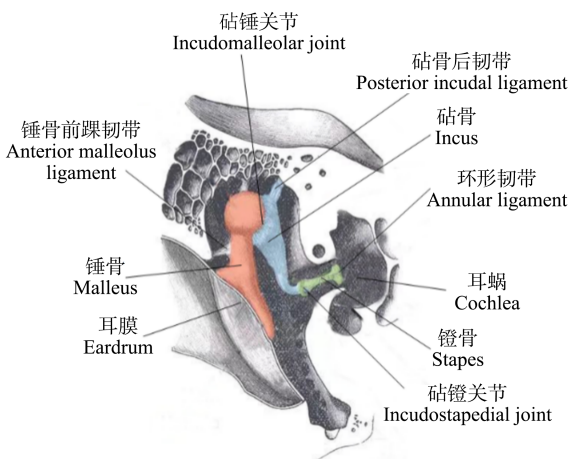


图1 中耳结构示意图

Fig. 1 Schematic diagram of the middle ear's structure

$$\left\{ \begin{array}{l} M_M \ddot{X}_M + K_1 X_M + K_3 (X_M - X_I) + \\ C_1 \dot{X}_M + C_2 \dot{X}_M + C_3 (\dot{X}_M - \dot{X}_I) + \\ K_2 X_M = P \cos [\Omega \tau + \gamma W(\tau)] \\ M_I \ddot{X}_I + K_3 (X_I - X_M) + K_4 X_I + \\ K_5 (X_I - X_S) + C_3 (\dot{X}_I - \dot{X}_M) + \\ C_4 \dot{X}_M + C_5 (\dot{X}_I - \dot{X}_S) = 0 \\ M_S \ddot{X}_S + K_5 (X_S - X_I) + K_6 X_S + \\ K_{71} X_S + K_{72} X_S^2 + k_{73} X_S^3 + C_6 \dot{X}_S + \\ C_5 (\dot{X}_S - \dot{X}_I) + C_7 \dot{X}_S = 0 \end{array} \right. \quad (3)$$

中耳系统的线性振动方程是不计非线性项、阻尼和激励的振动方程。根据线性振动方程可以获得系统的固有频率和正则矩阵。系统的线性振动方程为

$$\left\{ \begin{array}{l} M_M \ddot{X}_M + K_1 X_M + K_2 X_M + K_3 (X_M - X_I) = 0 \\ M_I \ddot{X}_I + K_3 (X_I - X_M) + K_4 X_I + K_5 (X_I - X_S) = 0 \\ M_S \ddot{X}_S + K_5 (X_S - X_I) + K_6 X_S + K_{71} X_S = 0 \end{array} \right. \quad (4)$$

计算得正则振型为

$$\mathbf{u} = \begin{bmatrix} u_1^{(1)} & u_1^{(2)} & u_1^{(3)} \\ u_2^{(1)} & u_2^{(2)} & u_2^{(3)} \\ u_3^{(1)} & u_3^{(2)} & u_3^{(3)} \end{bmatrix}$$

对整个系统引入正则坐标 $\mathbf{X} = \mathbf{u}\mathbf{y}$, 其中, $\mathbf{X} = [X_M \ X_I \ X_S]^T$, $\mathbf{y} = [y_1 \ y_2 \ y_3]^T$ 。

通过变换得到正则坐标下的非线性振动方程

$$\mathbf{M}_p \ddot{\mathbf{y}} + \mathbf{C}_p \dot{\mathbf{y}} + \mathbf{K}_p \mathbf{y} + \mathbf{F}_N(\mathbf{y}) = \mathbf{F}_e(\tau) \quad (5)$$

其中,

$$\left\{ \begin{array}{l} \mathbf{M}_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{C}_p = \begin{bmatrix} \xi_1 & 0 & 0 \\ 0 & \xi_2 & 0 \\ 0 & 0 & \xi_3 \end{bmatrix} \\ \mathbf{K}_p = \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ 0 & 0 & \omega_3^2 \end{bmatrix} \\ \mathbf{F}_N(\mathbf{y}) = \mathbf{u}\mathbf{F} = \begin{bmatrix} F_{N1}(y_1, y_2, y_3) \\ F_{N2}(y_1, y_2, y_3) \\ F_{N3}(y_1, y_2, y_3) \end{bmatrix} \\ \mathbf{F}_e(\tau) = \mathbf{u}\mathbf{P} = \begin{bmatrix} P_{e1}(\tau) \\ P_{e2}(\tau) \\ P_{e3}(\tau) \end{bmatrix} = \begin{bmatrix} P_1 \cos [\Omega \tau + \gamma W(\tau)] \\ P_2 \cos [\Omega \tau + \gamma W(\tau)] \\ P_3 \cos [\Omega \tau + \gamma W(\tau)] \end{bmatrix} \end{array} \right. \quad (6)$$

式中, $\mathbf{F}$ 、 $\mathbf{P}$ 分别为非线性项和激励。

$$\left\{ \begin{array}{l} \mathbf{F} = \begin{bmatrix} 0 \\ 0 \\ k_{72} x_S^2 + k_{73} x_S^3 \end{bmatrix} \\ \mathbf{P} = \begin{bmatrix} P \cos(\Omega \tau + \gamma W(\tau)) \\ 0 \\ 0 \end{bmatrix} \end{array} \right. \quad (7)$$

正则坐标下的非线性振动方程展开式为

$$\left\{ \begin{array}{l} \ddot{y}_1 + \xi_1 \dot{y}_1 + \omega_1^2 y_1 + \alpha_1 y_1^2 + \alpha_2 y_1^3 + \alpha_3 y_1 y_2 + \\ \alpha_4 y_1^2 y_2 + \alpha_5 y_2^2 + \alpha_6 y_1 y_2^2 + \alpha_7 y_2^3 + \\ \alpha_8 y_1 y_3 + \alpha_9 y_1^2 y_3 + \alpha_{10} y_2 y_3 + \\ \alpha_{11} y_1 y_2 y_3 + \alpha_{12} y_2^2 y_3 + \alpha_{13} y_3^2 + \\ \alpha_{14} y_1 y_3^2 + \alpha_{15} y_2 y_3^2 + \alpha_{16} y_3^3 = \\ P_1 \cos [\Omega \tau + \gamma W(\tau)] \\ \ddot{y}_2 + \xi_2 \dot{y}_2 + \omega_2^2 y_2 + \beta_1 y_1^2 + \beta_2 y_1^3 + \beta_3 y_1 y_2 + \\ \beta_4 y_1^2 y_2 + \beta_5 y_2^2 + \beta_6 y_1 y_2^2 + \beta_7 y_2^3 + \\ \beta_8 y_1 y_3 + \beta_9 y_1^2 y_3 + \beta_{10} y_2 y_3 + \\ \beta_{11} y_1 y_2 y_3 + \beta_{12} y_2^2 y_3 + \beta_{13} y_3^2 + \\ \beta_{14} y_1 y_3^2 + \beta_{15} y_2 y_3^2 + \beta_{16} y_3^3 = \\ P_2 \cos [\Omega \tau + \gamma W(\tau)] \\ \ddot{y}_3 + \xi_3 \dot{y}_3 + \omega_3^2 y_3 + \gamma_1 y_1^2 + \gamma_2 y_1^3 + \gamma_3 y_1 y_2 + \\ \gamma_4 y_1^2 y_2 + \gamma_5 y_2^2 + \gamma_6 y_1 y_2^2 + \gamma_7 y_2^3 + \\ \gamma_8 y_1 y_3 + \gamma_9 y_1^2 y_3 + \gamma_{10} y_2 y_3 + \\ \gamma_{11} y_1 y_2 y_3 + \gamma_{12} y_2^2 y_3 + \gamma_{13} y_3^2 + \\ \gamma_{14} y_1 y_3^2 + \gamma_{15} y_2 y_3^2 + \gamma_{16} y_3^3 = \\ P_3 \cos [\Omega \tau + \gamma W(\tau)] \end{array} \right. \quad (8)$$

2 $\Omega \approx \omega_1$ 时窄带激励下第 1 阶主共振理论分析

多尺度法经过不断地完善,已经成为一种十分有效的近似计算方法,后来逐渐推广到窄带随机激励下的振动求解^[11]。当外界激励的频率接近系统的固有频率时会导致主共振的产生,即便是很小的激励幅值也能引起强烈的共振^[12]。人类中耳系统符合弱非线性系统的运动规律,因此在阻尼项、外激励及非线性项前冠以小参数 ε 。相关文献表明^[13],不考虑系统中内共振的影响,多自由度系统可以分成多个单自由度系统分别求解。式(8)中的第 1 式为耳垂骨振动微分方程,第 2 式为砧骨振动微分方程,第 3 式为镫骨振动微分方程。当 $\Omega = \omega_1 + \varepsilon \sigma_1$ 时,系统发生第 1 阶主共振, ω_1 为中耳系统第 1 阶固有频率,并引入小参数 ε 和调谐量 σ_1 对中耳系统第 1 阶主共振进行非线性分析。由于不考虑系统中内共振的影响,砧骨和镫骨的运动不会对中耳第 1 阶主共振的久期项产生贡献,因此可以将式(8)改写成

$$\ddot{y}_1 + \omega_1^2 y_1 = -\varepsilon \{ \xi_1 \dot{y}_1 + \alpha_1 y_1^2 + \alpha_2 y_1^3 + \alpha_3 y_1 y_2 + \\ \alpha_4 y_1^2 y_2 + \alpha_5 y_2^2 + \alpha_6 y_1 y_2^2 + \alpha_7 y_2^3 + \alpha_8 y_1 y_3 + \alpha_9 y_1^2 y_3 + \\ \alpha_{10} y_2 y_3 + \alpha_{11} y_1 y_2 y_3 + \alpha_{12} y_2^2 y_3 + \alpha_{13} y_3^2 + \alpha_{14} y_1 y_3^2 + \\ \alpha_{15} y_2 y_3^2 + \alpha_{16} y_3^3 - P_1 \cos [\Omega \tau + \gamma W(\tau)] \} \quad (9)$$

采用 T_0 、 T_1 两个时间尺度,设方程(9)的解如下:

$$y_1(\tau, \varepsilon) = y_{10}(T_0, T_1) + \varepsilon y_{11}(T_0, T_1) \quad (10)$$

式中, $T_0 = \tau$, $T_1 = \varepsilon T_0$ 。

记导数算子为 $D_0 = \frac{\partial}{\partial T_0}$, $D_1 = \frac{\partial}{\partial T_1}$ 。将式(10)代

入式(9), 展开后令两边 ε 的同次幂系数相同, 得到一组偏微分方程为

$$\varepsilon^0: D_0^2 y_{10} + \omega_1^2 y_{10} = 0 \quad (11)$$

$$\begin{aligned} \varepsilon^1: D_0^2 y_{11} + \omega_1^2 y_{11} = & -\{2D_0 D_1 y_{10} + \xi_1 \dot{y}_{10} + \alpha_1 y_{10}^2 + \\ & \alpha_2 y_{10}^3 + \alpha_3 y_{10} y_{20} + \alpha_4 y_{10}^2 y_{20} + \alpha_5 y_{20}^2 + \alpha_6 y_{10} y_{20}^2 + \\ & \alpha_7 y_{20}^3 + \alpha_8 y_{10} y_{30} + \alpha_9 y_{10}^2 y_{30} + \alpha_{10} y_{20} y_{30} + \\ & \alpha_{11} y_{10} y_{20} y_{30} + \alpha_{12} y_{20}^2 y_{30} + \alpha_{13} y_{30}^2 + \alpha_{14} y_{10} y_{30}^2 + \\ & \alpha_{15} y_{20} y_{30}^2 + \alpha_{16} y_{30}^3 - P_1 \cos[\Omega\tau + \gamma W(\tau)]\} \end{aligned} \quad (12)$$

方程(11)的解为

$$y_{10} = A_1(T_1)e^{i\omega_1 T_0} + \bar{A}_1(T_1)e^{-i\omega_1 T_0} \quad (13)$$

$$A_1 = \frac{a_1(T_1)}{2} e^{i\theta_1(T_1)}, \bar{A}_1 = \frac{a_1(T_1)}{2} e^{-i\theta_1(T_1)} \quad (14)$$

式中, a_1 为第1阶模态振幅; θ_1 为对应的初相位。

将式(13)代入式(12)中可得消除久期项的条件为

$$(2i\omega_1 D_1 A_1 + \xi_1 i\omega_1 A_1 + 3\alpha_2 A_1^2 \bar{A}_1) - \frac{P_1}{2} e^{i[\sigma_1 T_1 + \gamma W(\tau)]} = 0 \quad (15)$$

把式(14)代入式(15)分离实部和虚部得

$$\begin{cases} D_1 a_1 = -\frac{\xi_1 a_1}{2} + \frac{P_1}{2\omega_1} \sin \varphi_1 \\ a_1 D_1 \varphi_1 = \sigma_1 a_1 - \frac{3\alpha_2}{8\omega_1} a_1^3 + \frac{P_1}{2\omega_1} \cos \varphi_1 + a_1 \gamma W'(T_1) \end{cases} \quad (16)$$

式中, $\varphi_1 = \sigma_1 T_1 + \gamma W(T_1) - \theta_1$ 。

3 $\gamma = 0$ 时系统运动

先考虑 $\gamma = 0$ 时的情况, 此时式(16)变为确定性系统:

$$\begin{cases} D_1 a_1 = -\frac{\xi_1 a_1}{2} + \frac{P_1}{2\omega_1} \sin \varphi_1 \\ a_1 D_1 \varphi_1 = \sigma_1 a_1 - \frac{3\alpha_2}{8\omega_1} a_1^3 + \frac{P_1}{2\omega_1} \cos \varphi_1 \end{cases} \quad (17)$$

$D_1 a_1 = 0, D_1 \varphi_1 = 0$ 可以得到稳态系统的幅频响应方程:

$$\left(-\frac{\xi_1 a_1}{2}\right)^2 + \left(\sigma_1 a_1 - \frac{3\alpha_2}{8\omega_1} a_1^3\right)^2 = \left(\frac{P_1}{2\omega_1}\right)^2 \quad (18)$$

根据 RUSINEK R 等^{[14]2609-2621} 在研究中得出的人体中耳系统各物理参数, 给各参数赋值, 其中 $\varepsilon = 0.1$ 。

图3是根据文献^{[14]2609-2621}中的数据得到中耳系统第1阶主共振的幅频响应曲线。其中外部激励的幅值 $P_0 = 0.27$ 。对比分析发现, 中耳系统对外部激励的变化比较敏感, 随着激励幅值的增大, 振幅会出现“跳跃现象”。

为了研究外部声波激励对中耳系统的具体影响, 绘制了系统关于参数外部激励幅值的分岔图和最大 Lyapunov 指数图如图4、图5所示。图4为系统响应随激励幅值 P 变化的分岔图。由图4可以看出, 随着外部激励幅值的增大, 系统响应会出现混沌响应特性。

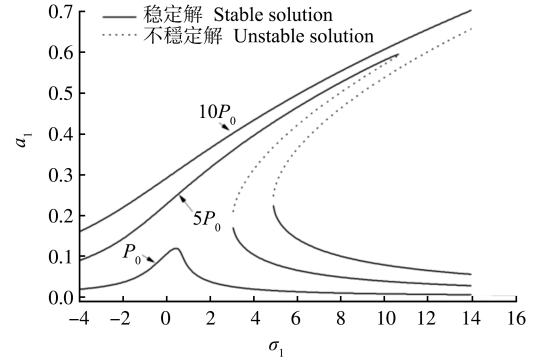


图3 幅频响应曲线

Fig. 3 Curves of amplitude-frequency response

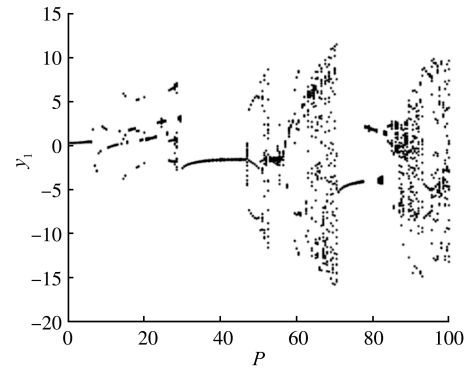


图4 激励 P 分岔图

Fig. 4 Bifurcation diagram of excitation P

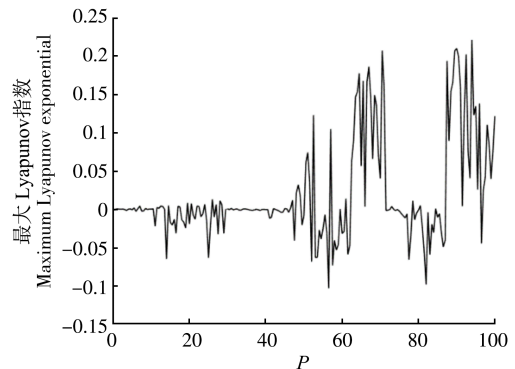


图5 最大 Lyapunov 指数

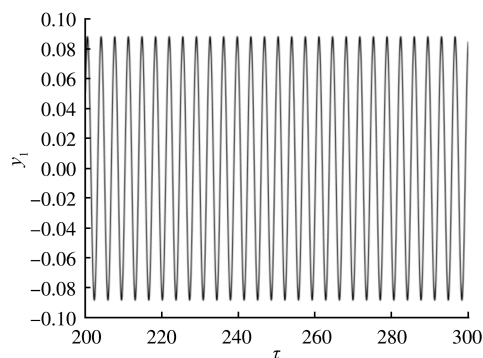
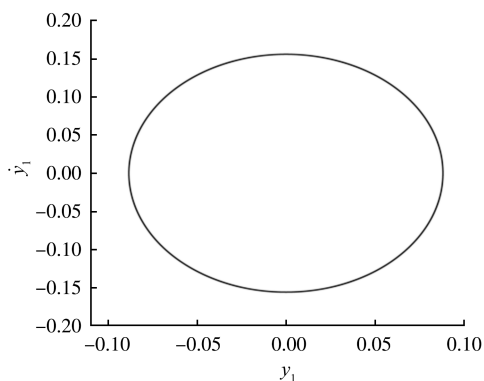
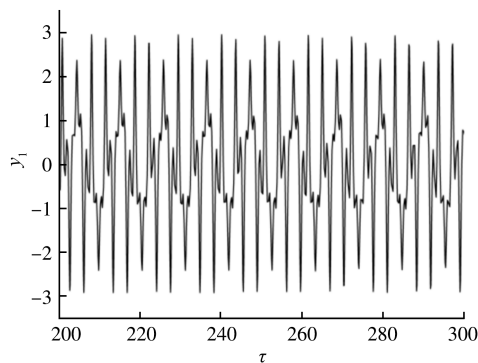
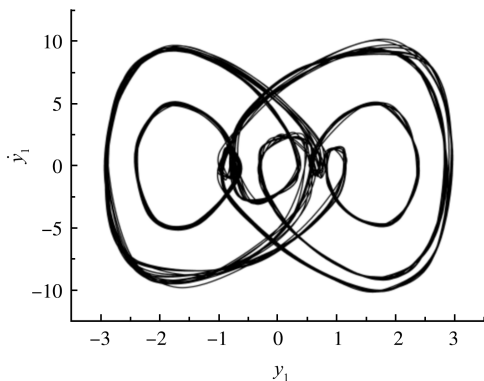
Fig. 5 Maximum Lyapunov exponential

由图5中系统的最大 Lyapunov 指数再次验证, 随着激励幅值的增加, 系统响应表现出混沌响应特性。

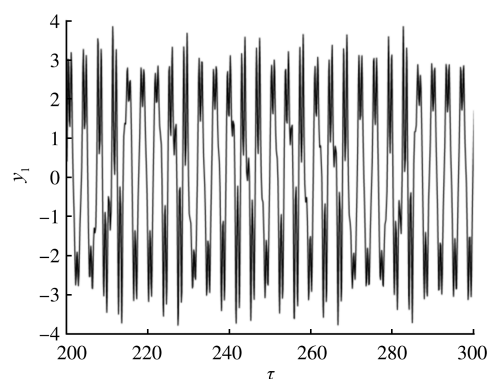
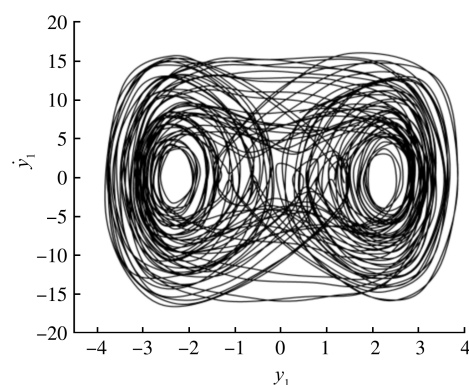
图6所示为激励幅值 $P = P_0$ 时中耳系统的响应, 图7为激励幅值 $P = P_0$ 时的相图。振动稳定时, 系统时间响应为简谐波, 相图为一圆。

图8所示为激励幅值 $P = 30$ 时中耳系统的时间响应, 图9为激励幅值 $P = 30$ 时的相图。振动稳定时, 系统响应呈现拟周期响应, 相图为具有一定宽度的曲线带。

图10、图11是当声波激励幅值 $P = 70$ 时系统响应和相图。此时系统呈现混沌响应的特征, 其相图为互

图 6 $P=P_0$ 时间响应曲线Fig. 6 Time response curve of $P=P_0$ 图 7 $P=P_0$ 相图Fig. 7 Phase plot of $P=P_0$ 图 8 $P=30$ 时间响应曲线Fig. 8 Time response curve of $P=30$ 图 9 $P=30$ 相图Fig. 9 Phase plot of $P=30$

相缠绕和交叉的曲线。

图 10 $P=70$ 时间响应曲线Fig. 10 Time response curve of $P=70$ 图 11 $P=70$ 相图Fig. 11 Phase plot of $P=70$

4 $\gamma \neq 0$ 时系统运动

当 $\gamma \neq 0$ 时,考虑了噪声对系统的影响,可设方程(16)的解如下:

$$a_1 = a_0 + a_{01}, \varphi_1 = \varphi_0 + \varphi_{01} \quad (19)$$

式中, a_0 和 φ_0 可由式(18)的解确定, a_{01} 和 φ_{01} 为摄动项。将方程(16)在 (a_0, φ_0) 处线性化,得到关于摄动项 a_{01} 和 φ_{01} 自治微分方程为

$$\begin{cases} a'_{01} = -\frac{\xi_1}{2} a_{01} + \left(\frac{3\alpha_2 a_0^3}{8\omega_1} - \sigma_1 a_0 \right) \varphi_{01} \\ \varphi'_{01} = -\left(\frac{9\alpha_2 a_0}{8\omega_1} - \frac{\sigma_1}{a_0} \right) a_{01} - \frac{\xi_1}{2} \varphi_{01} + \gamma W' \end{cases} \quad (20)$$

可写成对应的伊藤微分方程

$$\begin{cases} da_{01} = [Q_1 a_{01} - Q_2 \varphi_{01}] dT_1 \\ d\varphi_{01} = [Q_3 a_{01} - Q_4 \varphi_{01}] dT_1 + \gamma dW \end{cases} \quad (21)$$

式中, $Q_1 = -\frac{\xi_1}{2}$, $Q_2 = \sigma_1 a_0 - \frac{3\alpha_2 a_0^3}{8\omega_1}$, $Q_3 = \frac{\sigma_1}{a_0} - \frac{9\alpha_2 a_0}{8\omega_1}$, $Q_4 = \frac{\xi_1}{2}$ 。

根据矩法^[15]和伊藤公式求得 Ea_{01}^2 , $Ea_{01}\varphi_{01}$, $E\varphi_{01}^2$ 的值为

$$\begin{cases} Ea_{01}^2 = -\frac{Q_2^2 \gamma^2}{2(Q_2 Q_3 - Q_1 Q_4)(Q_1 - Q_4)} \\ Ea_{01} \varphi_{01} = -\frac{Q_1^2 \gamma^2}{2(Q_2 Q_3 - Q_1 Q_4)(Q_1 - Q_4)} \\ E\varphi_{01}^2 = -\frac{[Q_2 Q_3 + Q_1(Q_1 - Q_4)] \gamma^2}{2(Q_2 Q_3 - Q_1 Q_4)(Q_1 - Q_4)} \end{cases} \quad (22)$$

结合式(20)、式(22)可得系统2阶矩为

$$\begin{cases} Ea_1 = a_0 \\ Ea_1^2 = a_0^2 - \frac{Q_2^2 \gamma^2}{2(Q_2 Q_3 - Q_1 Q_4)(Q_1 - Q_4)} \\ E\varphi_1 = \varphi_0 \\ E\varphi_1^2 = \varphi_0^2 - \frac{[Q_2 Q_3 + Q_1(Q_1 - Q_4)] \gamma^2}{2(Q_2 Q_3 - Q_1 Q_4)(Q_1 - Q_4)} \end{cases} \quad (23)$$

式(23)给出了窄带随机激励幅值 P_0 作用下中耳系统第1阶主共振2阶矩 Ea_1^2 的表达式。图12所示为扰动强度 $\gamma = 0.05$ 时,窄带随机激励下第1阶主共振的均方响应曲线。对比图3中激励幅值为 P_0 的幅频特性曲线,两曲线有相同的拓扑结构。

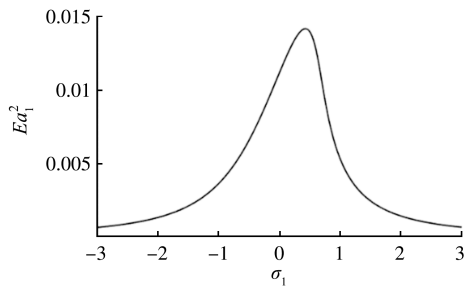


图12 主共振均方响应曲线

Fig. 12 Mean square response curve of the main resonance

为了研究窄带随机激励对中耳系统的影响,在激励幅值为 P_0 的条件下选取扰动强度 $\gamma = 0.05$ 和 $\gamma = 0.1$ 对中耳系统进行仿真分析。图13、图14分别为 $\gamma = 0.05$ 和 $\gamma = 0.1$ 时的中耳系统时间历程图。对比图5确定激励下时间历程图可知,受随机扰动 $\gamma W(\tau)$ 的影响,振动稳定时,系统的响应为拟周期响应,且扰动强度越大,系统时间响应波动范围越大。

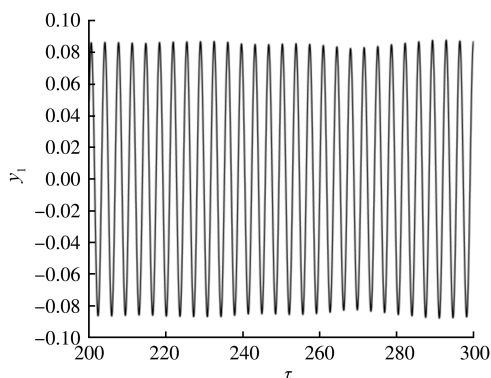


图13 $\gamma = 0.05$ 时间响应曲线

Fig. 13 Time response curve of $\gamma = 0.05$

图15、图16分别为 $\gamma = 0.05$ 和 $\gamma = 0.1$ 的相图。由图15、图16可知,随着有界窄带随机激励扰动强度 γ 的增大,系统第1阶主共振相图极限环变厚。

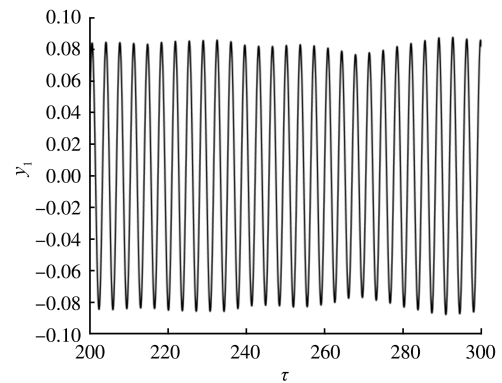


图14 $\gamma = 0.1$ 时间响应曲线

Fig. 14 Time response curve of $\gamma = 0.1$

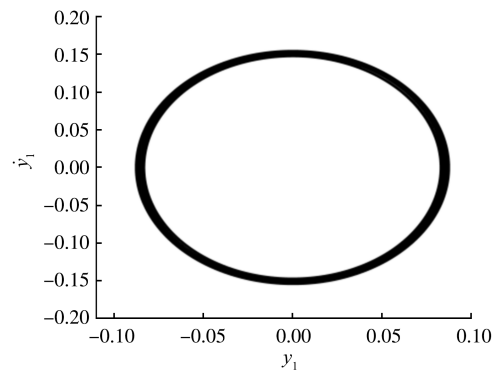


图15 $\gamma = 0.05$ 相图

Fig. 15 Phase plot of $\gamma = 0.05$

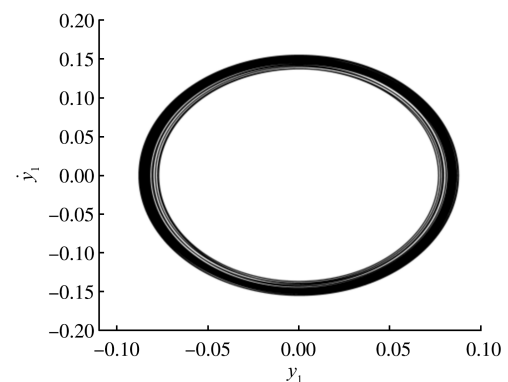


图16 $\gamma = 0.1$ 相图

Fig. 16 Phase plot of $\gamma = 0.1$

5 结论

应用多尺度法研究中耳系统在简谐激励和有界窄带随机激励下的响应。得到了第1阶主共振频响方程和窄带随机激励下系统的2阶矩。结果表明,当随机激励扰动强度 $\gamma = 0$ 时,不同的外部激励幅值会对系统产生不同的影响;随着激励幅值的增大,中耳系统的振幅出现“跳跃现象”,中耳系统表现出不同的

运动状态。当随机扰动强度 $\gamma \neq 0$ 时,系统的响应为拟周期响应,随着有界窄带随机激励扰动强度 γ 的增大,系统相图极限环变厚。

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